Operators
and
Matrices
Volume 14, Number 2 (2020), 515–521
doi:10.7153/oam-2020-14-36

SOME C^*–ALGEBRAS WHOSE EXT IS NOT A GROUP

FLORIN POP

Abstract. It has been known since 1977 that, for a unital, separable C^*-algebra A, $Ext_u(A)$ is a group if and only if every unital $*$-monomorphism of A with values in the Calkin algebra $\mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$ has a unital completely positive lifting to $\mathbb{B}(\ell^2)$. While Ext_u is a group for all C^*-algebras with the Local Lifting Property, the information in the opposite direction is rather scarce, with only two examples being known to this day whose Ext_u is *not* a group.

In this note we present several new examples of separable C^*-algebras whose Ext_u is not a group. These examples are a consequence of the existence of a finite dimensional operator system in the Calkin algebra whose identity map has no completely positive lifting to $B(\ell^2)$.

Mathematics subject classification (2010): 46L05, 46L06.
Keywords and phrases: C^*-algebra, Ext, completely positive map.

REFERENCES

[1] J. ANDERSON, *A C^*-algebra A for which $Ext(A)$ is not a group*, Ann. Math. **107** (1978), 455–458.
[2] W. B. ARVESON, *A note on essentially normal operators*, Proc. Roy. Irish Acad. sect.A **74** (1974), 143–146.
[3] W. B. ARVESON, *Notes on extensions of C^*-algebras*, Duke Math. J. **44** (1977), 329–355.
[4] L. BROWN, R. G. DOUGLAS, P. FILLMORE, *Unitary equivalence modulo the compact operators and extensions of C^*-algebras*, Proc. conf. operator theory, Lecture Notes in Math. **345**, 58–128, 1973.
[5] L. BROWN, R. G. DOUGLAS, P. FILLMORE, *Extensions of C^*-algebras and K-homology*, Ann. Math. **105** (1977), 265–324.
[6] M. D. CHOI, E. G. EFFROS, *The completely positive lifting problem for C^*-algebras*, Ann. Math. **104** (1976), 585–609.
[7] K. R. DAVIDSON, *C^*-algebras by example*, Fields Institute Monographs 6, American Mathematical Society, Providence, RI, 1996.
[8] U. HAAGERUP, S. THORBJORNSEN, *A new application of random matrices: $Ext(C^*_{red}(\mathbb{F}_2))$ is not a group*, Ann. Math. **162** (2005), 711–775.
[9] E. KIRCHBERG, *The Fubini theorem for exact C^*-algebras*, J. Operator Theory **10** (1983), 3–8.
[10] E. KIRCHBERG, *On nonsemisplit extensions, tensor products and exactness of group C^*-algebras*, Invent. Math. **112** (1993), 449–489.
[11] E. KIRCHBERG, *Commutants of unitaries in UHF algebras and functorial properties of exactness*, J. Reine Angew. Math. **452** (1994), 39–77.
[12] E. KIRCHBERG, S. WASSERMANN, *C^*-algebras generated by operator systems*, J. Funct. Anal. **155** (1998), 324–351.
[13] E. C. LANCE, *On nuclear C^*-algebras*, J. Funct. Anal. **12** (1973), 157–176.
[14] N. OZAWA, *About the QWEP conjecture*, Int. J. Math. **15** (2004), 501–530.
[15] V. I. PAULSEN, *Completely bounded maps and operator algebras*, Cambridge University Press, Cambridge (2003).
[16] G. PISIER, *Introduction to Operator Space Theory*, Cambridge University Press, Cambridge (2003).
[17] G. PISIER, *Tensor products of C^*-algebras and operator spaces, The Connes-Kirchberg problem*, Cambridge University Press (2020), to appear.
[18] F. POP, *Remarks on the completely bounded approximation property for C^*-algebras*, Houston J. Math. **43** (2017), 937–945.

Paper OaM-14-36

[19] D. VOICULESCU, *A non-commutative Weyl-von Neumann theorem*, Rev. Roum. Math. Pures Appl. **21** (1976), 97–113.

[20] S. WASSERMANN, *Tensor products of free group C^*-algebras*, Bull. London Math. Soc. **22** (1990), 375–380.