

SELF-ADJOINT EXTENSIONS OF THE TWO-VALLEY DIRAC OPERATOR WITH DISCONTINUOUS INFINITE MASS BOUNDARY CONDITIONS

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Abstract. We consider the four-component two-valley Dirac operator on a wedge in \mathbb{R}^2 with infinite mass boundary conditions, which enjoy a flip at the vertex. We show that it has deficiency indices $(1,1)$ and we parametrize all its self-adjoint extensions, relying on the fact that the underlying two-component Dirac operator is symmetric with deficiency indices $(0,1)$. The respective defect element is computed explicitly. We observe that there exists no self-adjoint extension, which can be decomposed into an orthogonal sum of two two-component operators. In physics, this effect is called *mixing the valleys*.

Mathematics subject classification (2010): 35P05, 35Q40, 81Q10.

Keywords and phrases: Dirac operator, infinite mass boundary condition, wedge, self-adjoint extensions, mixing the valleys.

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