

CONVERGENCE OF LAPLACIANS ON SMOOTH SPACES TOWARDS THE FRACTAL SIERPIŃSKI GASKET

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Abstract. The purpose of this article is to prove that – under reasonable assumptions – the canonical energy form on a graph-like manifold is quasi-unitarily equivalent with the energy form on the underlying discrete graph. Then we will apply this to approximate the standard energy form on the Sierpiński gasket by a family of energy forms on suitable graph-like manifolds.

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