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SPECTRAL OPTIMIZATION FOR SINGULAR SCHRÖDINGER OPERATORS

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Abstract. For several classes of singular Schrödinger operators which can be formally written as $-\Delta - \alpha\delta(x-\Gamma)$ we discuss the problem of optimization of their principal eigenvalue with respect to the shape of the interaction support Γ.

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