Operators and Matrices
Volume 14, Number 4 (2020), 777–793 doi:10.7153/oam-2020-14-49

AN EXTENSION OF THE BEURLING–CHEN–HADWIN–SHEN THEOREM FOR NONCOMMUTATIVE HARDY SPACES ASSOCIATED WITH FINITE VON NEUMANN ALGEBRAS

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Abstract. In 2015, Yanni Chen, Don Hadwin and Junhao Shen proved a noncommutative version of Beurling's theorems for a continuous unitarily invariant norm α on a tracial von Neumann algebra $(\mathscr{M},\tau)$, where α is $\|\cdot\|_1$-dominating with respect to τ. In the paper, we first define a class of norms $N_\Delta(\mathscr{M},\tau)$ on $\mathscr{M}$, called determinant, normalized, unitarily invariant continuous norms on $\mathscr{M}$. If $\alpha \in N_\Delta(\mathscr{M},\tau)$, then there exists a faithful normal tracial state ρ on $\mathscr{M}$ such that $\rho(x) = \tau(xg)$ for some positive $g \in L^1(\mathscr{Z},\tau)$ and the determinant of g is positive. For every $\alpha \in N_\Delta(\mathscr{M},\tau)$, we study the noncommutative Hardy spaces $H^\alpha(\mathscr{M},\tau)$, then prove that the Chen-Hadwin-Shen theorem holds for $L^\alpha(\mathscr{M},\tau)$. The key ingredients in the proof of our result include a factorization theorem and a density theorem for $L^\alpha(\mathscr{M},\rho)$.

Mathematics subject classification (2010): 46E20, 30H10, 30J99, 47L10.
Keywords and phrases: Gauge norm, von Neumann algebra, Beurling theorem.

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