

FACTORIZATION AND RANGE INCLUSION OF ADJOINTABLE OPERATORS ON THE WEIGHTED HILBERT C^* -MODULES

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Abstract. The indefinite inner products induced by invertible and self-adjoint weights are introduced for elements in Hilbert C^* -modules. The solvability of the equation $AX = C$ is considered for Hilbert C^* -module operators. Some equivalent conditions concerning two aspects of factorization and range inclusion are refined and generalized to the weighted case.

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REFERENCES

- [1] M. L. ARIAS, G. CORACH AND M. C. GONZALEZ, *Partial isometries in semi-Hilbertian spaces*, Linear Algebra Appl. **428**, 7 (2008), 1460–1475.
- [2] T. YA. AZIZOV AND I. S. IOKHVIDOV, *Linear operators in spaces with an indefinite metric*, Translated from the Russian by E. R. Dawson. Pure and Applied Mathematics, New York, A Wiley-Interscience Publication. John Wiley and Sons, Ltd., Chichester, 1989.
- [3] B. A. BARNES, *Majorization, range inclusion, and factorization for bounded linear operators*, Proc. Amer. Math. Soc. **133**, 1 (2005), 155–162.
- [4] R. G. DOUGLAS, *On majorization, factorization, and range inclusion of operators on Hilbert spaces*, Proc. Amer. Math. Soc. **17** (1966), 413–415.
- [5] M. EMBRY, *Factorization of operators on a Banach space*, Proc. Amer. Math. Soc. **38** (1973), 587–590.
- [6] X. FANG, M. S. MOSLEHIAN AND Q. XU, *On majorization and range inclusion of operators on Hilbert C^* -modules*, Linear Multilinear Algebra **66** (2018), 2493–2500.
- [7] X. FANG, J. YU AND H. YAO, *Solutions to operator equations on Hilbert C^* -modules*, Linear Algebra Appl. **431** (2009), 2142–2153.
- [8] L. JAKÓBCZYK, *Borchers algebra formulation of an indefinite inner product quantum field theory*, J. Math. Phys. **25**, 3 (1984), 617–622.
- [9] E. C. LANCE, *Hilbert C^* -modules—A toolkit for operator algebraists*, Cambridge University Press, Cambridge, 1995.
- [10] M. S. MOSLEHIAN AND M. DEGHANI, *Operator convexity in Krein spaces*, New York J. Math. **20** (2014), 133–144.
- [11] M. S. MOSLEHIAN, M. KIAN AND Q. XU, *Positivity of 2×2 block matrices of operators*, Banach J. Math. Anal. **13**, 3 (2019), 726–743.
- [12] L. RODMAN, *A note on indefinite Douglas’ lemma*, Operator theory in inner product spaces, 225–229, Oper. Theory Adv. Appl. **175**, Birkhäuser, Basel, 2007.
- [13] J. L. ŠMUL’JAN, *Non-expanding operators in a finite-dimensional space with indefinite metric* (Russian), Uspehi Mat. Nauk **18**, 6 (114) (1963), 225–230.
- [14] A. STROHMAIER, *On noncommutative and pseudo-Riemannian geometry*, J. Geom. Phys. **56**, 2 (2006), 175–195.
- [15] Q. XU, Y. CHEN AND C. SONG, *Representations for weighted Moore–Penrose inverses of partitioned adjointable operators*, Linear Algebra Appl. **438**, 1 (2013), 10–30.

- [16] Q. XU AND X. FANG, *A note on majorization and range inclusion of adjointable operators on Hilbert C^* -modules*, *Linear Algebra Appl.* **516** (2017), 118–125.