

ON A FUNCTIONAL CALCULUS FOR UNITARY OPERATORS IN PONTRYAGIN SPACES

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Abstract. This work is devoted to a functional description of the weakly closed algebra generated by a Pontryagin Space unitary operator under the condition that this operator contains a part like the two-side shift operator. The latter means that the inverse operator does not belong to the algebra.

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