

TOPOLOGICAL PROPERTIES OF THE BLOCK NUMERICAL RANGE OF OPERATOR MATRICES

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Abstract. We show that the block numerical range of an $n \times n$ -operator matrix \mathcal{A} corresponding to an operator A on the Banach space X with respect to a decomposition $X = \prod X_j$ has at most n connected components. We then characterize operator matrices with finite block numerical range. As an important tool we prove an inclusion theorem for the block numerical ranges of the principal submatrices of \mathcal{A} .

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