

INTEGRAL REPRESENTATIONS OF SOME FAMILIES OF OPERATOR MONOTONE FUNCTIONS

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Abstract. We obtain an integral representation of holomorphic function $P_\alpha(z)$ which is real on the positive part of the real axis and formed

$$P_\alpha(x) = \left(\frac{x^\alpha + 1}{2} \right)^{\frac{1}{\alpha}} \quad (x \geq 0).$$

For this purpose we define a two variable function which is substituted for an argument θ , and also find an explicit real and imaginary part of $P_\alpha(x + iy)$.

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