

## ADDITIVE LOCAL MULTIPLICATIONS AND ZERO-PRESERVING MAPS ON $C(X)$

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**Abstract.** Suppose  $X$  is a compact Hausdorff space. In terms of topological properties of  $X$ , we find topological conditions on  $X$  that are equivalent to each of the following: 1. Every additive local multiplication on  $C(X)$  is a multiplication, 2. Every additive local multiplication on  $C_R(X)$  is a multiplication, 3. Every additive map on  $C(X)$  that is zero-preserving (i.e.,  $f(x) = 0$  implies  $(Tf)(x) = 0$ ) has the form  $T(f) = T(1)\operatorname{Re}f + T(i)\operatorname{Im}f$ .

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