

ISOMORPHISMS OF $BV(\sigma)$ SPACES

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Abstract. In this paper we investigate the relationship between the properties of a compact set $\sigma \subseteq \mathbb{C}$ and the structure of the space $BV(\sigma)$ of functions of bounded variation (in the sense of Ashton and Doust) defined on σ . For the subalgebras of absolutely continuous functions on σ , it is known that for certain classes of compact sets one obtains a Gelfand–Kolmogorov type result: the function spaces $AC(\sigma_1)$ and $AC(\sigma_2)$ are isomorphic if and only if the domain sets σ_1 and σ_2 are homeomorphic. Our main theorem is that in this case the isomorphism must extend to an isomorphism of the $BV(\sigma)$ spaces. An application is given to the spectral theory of $AC(\sigma)$ operators.

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