

LVOV–KAPLANSKY CONJECTURE ON UT_m^+ WITH THE TRANSPOSE INVOLUTION

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Abstract. Let UT_m be the algebra of all $m \times m$ upper matrices with entries in a field F . Let us consider UT_m equipped with the transpose involution $*$. Under a mild technical assumption on F , we will show that the image of any multilinear Jordan polynomial in three variables evaluated on $UT_m^+ = \{U \in UT_m \mid U^* = U\}$ is a vector space. In particular, we will determine a basis for such image. As an application, we will describe the set of values of some multilinear Jordan polynomials in four variables.

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