

UNIFORMLY EXPONENTIAL DICHOTOMY FOR STRONGLY CONTINUOUS QUASI GROUPS

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Abstract. A strongly continuous quasi group (C_0 -quasi group) is established as an extension of a C_0 -quasi semigroup on a Banach space. The fundamental properties of the C_0 -quasi groups are derived from the properties of C_0 -quasi semigroups. It is identified a sufficient condition for an infinitesimal generator of a C_0 -quasi group. The infinitesimal generator of a C_0 -quasi group generates a non-autonomous the abstract Cauchy problem that is well-posed. Uniformly exponential stability of the C_0 -quasi groups and the C_0 -quasi semigroups on a Banach space X can be identified by the associated evolution semigroups on the spaces $L_p(\mathbb{R}, X)$ and $L_p(\mathbb{R}^+, X)$, $1 \leq p < \infty$, respectively. The sufficient and necessary conditions, called Dichotomy Theorem, for the uniformly exponential dichotomy of the C_0 -quasi groups and the C_0 -quasi semigroups are characterized by the associated evolution semigroups. The hyperbolicity of the evolution semigroups is used in the characterization. Dichotomy Theorem can also be identified by a Green's function induced by the associated evolution semigroup. Moreover, the infinitesimal generator of the associated evolution semigroup becomes the main subject in establishment of the sufficiency and necessity for the uniformly exponential stability of the C_0 -quasi semigroups.

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