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NONLINEAR LIE DERIVATIONS OF INCIDENCE ALGEBRAS

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Abstract. Let $(X,\leqslant)$ be a locally finite preordered set and R a 2-torsion free commutative ring with unity, $I(X,R)$ the incidence algebra of X over R. In this paper, we give an explicit description of the structure of nonlinear Lie derivations of $I(X,R)$. We prove that every nonlinear Lie derivation of $I(X,R)$ is a sum of an inner derivation, a transitive induced derivation and an additive induced Lie derivation.

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