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REFINING EIGENVALUE INEQUALITIES FOR BLOCK 2×2 POSITIVE SEMIDEFINITE MATRICES

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Abstract. In this paper, by employing a result due to Bourin, Lee and Lin for block 2×2 positive semidefinite matrices, and by using gradients of Gateaux differentiable G-increasing functions, we show refinements of some majorization inequality by Lin and Wolkowicz for the eigenvalues of these block matrices. In particular, we establish a refinement for 2×2 version of Hiroshima's inequality.

We also consider some special cases of the obtained result.

Mathematics subject classification (2020): 15A18, 15A42, 15A45, 15A60.

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