

A NON-UNITAL GENERALIZED TRACE AND LINEAR COMPLEX STRUCTURES

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Abstract. A basis-free formula for the generalized trace of a linear map between tensor products of vector spaces is proposed which does not refer to scalar multiplication or scalar valued functions. The main application is to real vector spaces with complex structure operators.

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