Operators
and
Matrices
Volume 15, Number 2 (2021), 677–684
doi:10.7153/oam-2021-15-45

THE ALGEBRA GENERATED BY SIMPLE ELEMENTS OF A MATRIX CENTRALIZER

RALPH JOHN DE LA CRUZ

Abstract. Let $\mathscr{C}(S)$ denote the centralizer of an arbitrary square matrix S. An element $A \in \mathscr{C}(S)$ is simple if $A-I$ is of rank 1. Let $\mathscr{A}_S$ denote the subalgebra generated by the simple elements of $\mathscr{C}(S)$. We use the Weyr canonical form to describe the subalgebra $\mathscr{A}_S$, and we show that if $\lambda_1,\ldots,\lambda_k$ are the distinct eigenvalues of S, and l is the number of defective eigenvalues of S, then $\mathscr{A}_S$ is of dimension $l+\sum_{i=1}^k \operatorname{nullity}(S-\lambda_i I)^2$.

Mathematics subject classification (2020): 15A03, 15A27.
Keywords and phrases: Simple, Weyr structure, Weyr canonical form, centralizer, defective.

REFERENCES

[1] T. AGAPITO, A. T. PARAS, *ψ_S-orthogonal matrices and ψ_S-symmetries*, Linear Algebra Appl. **584** (2020) 185–196.

[2] R. J. DE LA CRUZ, K. DELA ROSA, D. I. MERINO, A. T. PARAS, *The Cartan-Diedonne-Scherk Theorems for complex S-orthogonal matrices*, Linear Algebra Appl. **458** (2014) 251–260.

[3] M. CATRAL, L. LEBTAHI, J. STUART, N. THOME, *On a matrix group constructed from an $\{R,s+1,k\}$-potent matrix*, Linear Algebra Appl. **461** (2014) 200–210.

[4] R. J. DE LA CRUZ, D. I. MERINO, A. T. PARAS, *S orthogonal matrices and S symmetries*, Linear Algebra Appl. **474** (2015) 213–229.

[5] J. DIEUDONNE, *Sur les generateurs des groupes classiques*, Summa Bras. Math. **3** (1955) 149–178.

[6] R. A. HORN AND C. R. JOHNSON, *Topics in Matrix Analysis*, Cambridge Univesity Press, New York, 1991.

[7] G. HUANG, F. YIN, *Constrained inverse eigenproblem and associated approximation problem for anti-Hermitian R-symmetric matrices*, Applied Mathematics and Computation **186** (2007) 426–434.

[8] G. HUANG, F. YIN, *Matrix inverse problem and its optimal approximation problem for R-symmetric matrices*, Applied Mathematics and Computation **189** (2007) 482–489.

[9] K. O'MEARA, J. CLARK, C. VINSONHALER, *Advanced topics in linear algebra: weaving matrix problems through the Weyr form*, Oxford University Press, Oxford 2011.

[10] H. RADJAVI, *Decomposition of matrices into simple involutions*, Linear Algebra Appl., **12** (1975) 247–255.

[11] P. SCHERK, *On the decomposition of orthogonalities into symmetries*, Proc. Amer. Math. Soc. **1** (1950) 481–491.

[12] W. TRENCH, *Characterization and properties of matrices with generalized symmetry or skew symmetry*, Linear Algebra Appl. **377** (2004) 207–218.

[13] W. TRENCH, *Minimization problems for (R,S)-symmetric and (R,S)-skew symmetric matrices*, Linear Algebra Appl. **389** (2004) 23–31.

[14] F. UHLIG, *Constructive ways for generating (generalized) real orthogonal matrices as products of (generalized) symmetries*, Linear Algebra Appl. **332–334** (2001) 459–467.