

## ON THE COMPACTNESS AND SPECTRA OF THE GENERALIZED DIFFERENCE OPERATOR ON THE SPACES $\ell^\infty$ AND $\text{bv}$

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*Abstract.* In this paper, we consider the compactness and the spectrum of the generalized difference operator  $\Delta_{ab}$  on the Banach sequence spaces  $\ell^\infty$ , of bounded sequences, and  $\text{bv}$ , of bounded variation sequences, which allows us to generalize and extend some existing results for the operator  $\Delta_{ab}$ . Furthermore, the results for the operator  $\Delta_{ab}$  on  $\ell^\infty$  are new even in the case where  $\Delta_{ab}$  is reduced to the difference operator  $\Delta$ .

*Mathematics subject classification (2020):* 47A10, 47B37, 46B45.

*Keywords and phrases:* Spectrum, difference operators, sequence spaces.

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