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ON m–QUASI–TOTALLY–(α,β)–NORMAL OPERATORS

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Abstract. An operator $\mathscr{S}$ acting on a Hilbert space is called m-quasi-totally-(α,β)-normal $(0\leqslant\alpha\leqslant 1\leqslant\beta)$ if

$$\alpha^2\mathscr{S}^{m*}(\mathscr{S}-\lambda)^*(\mathscr{S}-\lambda)\mathscr{S}^m\leqslant\mathscr{S}^{m*}(\mathscr{S}-\lambda)(\mathscr{S}-\lambda)^*\mathscr{S}^m\leqslant\beta^2\mathscr{S}^{m*}(\mathscr{S}-\lambda)^*(\mathscr{S}-\lambda)\mathscr{S}^m$$

for a natural number m and for all $\lambda\in\mathbb{C}$. m-quasi-totally-(α,β)-normal operator is equivalent to the study of mutual majorization between $(\mathscr{S}-\lambda)\mathscr{S}^m$ and $(\mathscr{S}-\lambda)^*\mathscr{S}^m$ for a natural number m and for all $\lambda\in\mathbb{C}$. This article aims to establish various inequalities between the operator norm and the numerical radius of m-quasi-totally-(α,β)-normal operators in Hilbert spaces. Further, this article analyzes spectral and algebraic properties of m-quasi-totally-(α,β)-normal operators.

Mathematics subject classification (2020): 47B15, 47B20, 47A15.
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