

## APPROXIMATION OF THE NUMERICAL RANGE OF POLYNOMIAL OPERATOR MATRICES

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*Abstract.* A linear operator on a Hilbert space may be approximated by finite matrices choosing an orthonormal basis of the Hilbert space. In this paper we establish an approximation of the  $q$ -numerical range of a bounded and an unbounded polynomial operator by variational methods. Applications to Hain-Lüst operator and Stokes operator are also given.

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