Operators
and
Matrices
Volume 15, Number 3 (2021), 1089–1108 doi:10.7153/oam-2021-15-68

ON SEMIMONOTONE STAR MATRICES AND LINEAR COMPLEMENTARITY PROBLEM

R. JANA *, A. K. DAS, S. SINHA

Abstract. In this article, we introduce the class of semimonotone star (E_0^s) matrices. We establish the importance of the class of E_0^s-matrices in the context of complementarity theory. We show that the principal pivot transform of an E_0^s-matrix is not necessarily E_0^s in general. However, we prove that $\tilde{E}_0^s$-matrices, a subclass of the E_0^s-matrices with some additional conditions, is fully semimonotone matrix by showing this class is in P_0. We prove that LCP(q,A) can be processable by Lemke's algorithm if $A \in \tilde{E}_0^s \cap P_0$. We find some conditions for which the solution set of LCP(q,A) is bounded and stable under the $\tilde{E}_0^s$-property. We propose an algorithm based on an interior point method to solve LCP(q,A) given $A \in \tilde{E}_0^s$.

Mathematics subject classification (2020): 90C33, 90C90, 15B99.

Keywords and phrases: Linear complementarity problem, principal pivot transform, Lemke's algorithm, interior point method, semimonotone star matrix, $\tilde{E}_0^s$-matrix.

REFERENCES

[1] ILAN ADLER, RICHARD W. COTTLE AND SUSHIL VERMA, *Sufficient matrices belong to L*, Mathematical Programming 106 (2) (2006), pp. 391–401.

[2] M. AGANAGIC AND R. W. COTTLE, *A constructive characterization of Q_0-matrices with nonnegative principal minors*, Mathematical Programming, 37 (2) (1987), pp. 223–231.

[3] M. CAO AND M. C. FERRIS, *P_c-matrices and the linear complementarity problem*, Linear Algebra and its Applications, 246 (1996), pp. 299–312.

[4] T. H. CHU, *On semimonotone matrices with nonnegative principal minors*, Linear algebra and its applications, 367 (2003), pp. 147–154.

[5] RICHARD W. COTTLE, *A field guide to the matrix classes found in the literature of the linear complementarity problem*, Journal of Global Optimization 46 (4) (2010), pp. 571–580.

[6] R. W. COTTLE, *The principal pivoting method revisited*, Mathematical Programming, 48 (1) (1990), pp. 369–385.

[7] R. W. COTTLE AND S. M. GUU, *Two characterizations of sufficient matrices*, Linear algebra and its applications, 170 (1992), pp. 65–74.

[8] R. W. COTTLE, J. S. PANG AND R. E. STONE, *The linear complementarity problem*, Society for Industrial and Applied Mathematics, 2009.

[9] RICHARD W. COTTLE AND RICHARD E. STONE, *On the uniqueness of solutions to linear complementarity problems*, Mathematical Programming 27 (2) (1983), pp. 191–213.

[10] A. K. DAS, *Properties of some matrix classes based on principal pivot transform*, Annals of Operations Research, 243 (1–2) (2016), pp. 375–382.

[11] A. K. DAS, R. JANA AND DEEPMALA, *On Generalized Positive Subdefinite Matrices and Interior Point Algorithm*, International Conference on Frontiers in Optimization: Theory and Applications, pp. 3–16, 2016.

[12] B. C. EAVES, *The linear complementarity problem*, Management science, 17 (9) (1971), pp. 612–634.

[13] F. FLORES-BAZÁN AND R. LÓPEZ, *Characterizing Q-matrices beyond L-matrices*, Journal of optimization theory and applications, 127 (2) (2005), pp. 447–457.

[14] M. S. GOWDA, *Pseudomonotone and copositive star matrices*, Linear Algebra and Its Applications, 113 (1989), pp. 107–118.

© ELEMENT, Zagreb
Paper OaM-15-68

[15] M. S. GOWDA AND J. S. PANG, *On solution stability of the linear complementarity problem*, Mathematics of Operations Research, 17 (1) (1992), pp. 77–83.

[16] R. JANA, A. K. DAS AND S. SINHA, *On Processability of Lemke's Algorithm*, Applications & Applied Mathematics 13 (2) (2018), pp. 1123–1131.

[17] C. JONES AND M. S. GOWDA, *On the connectedness of solution sets in linear complementarity problems*, Linear algebra and its applications, 272 (1–3) (1998), pp. 33–44.

[18] MASAKAZU KOJIMA, NIMROD MEGIDDO, TOSHIHITO NOMA, AND AKIKO YOSHISE, *A unified approach to interior point algorithms for linear complementarity problems*, Springer Science & Business Media 538 (1991).

[19] S. R. MOHAN, S. K. NEOGY AND A. K. DAS, *More on positive subdefinite matrices and the linear complementarity problem*, Linear Algebra and its Applications 338 (1–3) (2001), pp. 275–285.

[20] S. R. MOHAN, S. K. NEOGY AND A. K. DAS, *On the classes of fully copositive and fully semimonotone matrices*, Linear Algebra and its Applications 323 (1–3) (2001), pp. 87–97.

[21] G. S. R. MURTHY AND T. PARTHASARATHY, *Some Properties of Fully Semimonotone, Q_0-Matrices*, SIAM Journal on Matrix Analysis and Applications 16 (4) (1995), pp. 1268–1286.

[22] S. K. NEOGY AND A. K. DAS, *On almost type classes of matrices with Q-property*, Linear and Multilinear Algebra 53 (4) (2005), pp. 243–257.

[23] S. K. NEOGY AND A. K. DAS, *Principal pivot transforms of some classes of matrices*, Linear algebra and its applications 400 (2005), pp. 243–252.

[24] S. K. NEOGY AND A. K. DAS, *Some properties of generalized positive subdefinite matrices*, SIAM journal on matrix analysis and applications 27 (4) (2006), pp. 988–995.

[25] S. K. NEOGY, A. K. DAS AND ABHIJIT GUPTA, *Generalized principal pivot transforms, complementarity theory and their applications in stochastic games*, Optimization Letters 6 (2) (2012), pp. 339–356.

[26] J. S. PANG, *Iterative descent algorithms for a row sufficient linear complementarity problem*, SIAM journal on matrix analysis and applications, 12 (4) (1991), pp. 611–624.

[27] T. PARTHASARATHY AND B. SRIPARNA, *On the solution sets of linear complementarity problems*, SIAM Journal on Matrix Analysis and Applications, 21 (4) (2000), pp. 1229–1235.

[28] M. J. TODD AND Y. YE, *A centered projective algorithm for linear programming*, Mathematics of Operations Research, 15 (3) (1990), pp. 508–529.

[29] MICHAEL J. TSATSOMEROS AND MEGAN WENDLER, *Semimonotone Matrices*, Linear Algebra and its Applications, 2019.

[30] H. VÄLIAHO, *Almost copositive matrices*, Linear Algebra and its applications, 116 (1989), pp. 121–134.

[31] J. VON NEUMANN, *A certain zero-sum two-person game equivalent to the optimal assignment problem*, Contributions to the Theory of Games, 2 (1953), pp. 5–12.

[32] WALLACE C. PYE, *Almost P_0-matrices and the class Q*, Mathematical programming 57 (1–3) (1992), pp. 439–444.

Operators and Matrices
www.ele-math.com
oam@ele-math.com