

## STRICT SINGULARITY OF WEIGHTED COMPOSITION OPERATORS ON DERIVATIVE HARDY SPACES

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*Abstract.* We prove that the weighted composition operator  $W_{\phi, \varphi}$  fixes an isomorphic copy of  $\ell^p$  if the operator  $W_{\phi, \varphi}$  is not compact on the derivative Hardy space  $S^p$ . In particular, this implies that the strict singularity of the operator  $W_{\phi, \varphi}$  coincides with the compactness of it on  $S^p$ . Moreover, when  $p \neq 2$ , we characterize the conditions for those weighted composition operators  $W_{\phi, \varphi}$  on  $S^p$  which fix an isomorphic copy of  $\ell^2$ .

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