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GENERALIZED NUMERICAL RADIUS AND RELATED INEQUALITIES

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Abstract. In [2], Abu Omar and Kittaneh defined a new generalization of the numerical radius. That is, given a norm $N(\cdot)$ on $\mathscr{B}(H)$, the space of bounded linear operators over a Hilbert space H, and $A \in \mathscr{B}(H)$

$$w_N(A) = \sup_{\theta \in \mathbb{R}} N(Re(e^{i\theta}A)).$$

They proved several properties and introduced some inequalities. We continue with the study of this generalized numerical radius and we develop diverse inequalities involving w_N. We also study particular cases when $N(\cdot)$ is the p-Schatten norm with $p > 1$.

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Paper OaM-15-81

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