

## THE A-MODEL WITH MUTUALLY EQUAL MODEL PARAMETERS CAN LEAD TO A HILBERT SPACE MODEL

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*Abstract.* It is known that the A-model for higher order singular perturbations can be considered as a Hilbert space model if the model parameters are mutually distinct, and that it is necessarily a Pontryagin space model if otherwise. In this note we demonstrate that the A-model with mutually equal model parameters can nonetheless lead to a Hilbert space model if the extensions in the model space are instead described by suitable linear relations.

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