

PATH-CONNECTED CLOSURE OF UNITARY ORBITS

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Abstract. If \mathcal{A} and \mathcal{B} are unital C^* -algebras and $\pi : \mathcal{A} \rightarrow \mathcal{B}$ is a unital $*$ -homomorphism, then $\mathcal{U}_{\mathcal{B}}(\pi)^-$ is the set of all $*$ -homomorphisms from \mathcal{A} to \mathcal{B} that are approximately (unitarily) equivalent to π . We address the question of when $\mathcal{U}_{\mathcal{B}}(\pi)^-$ is path-connected with respect to the topology of pointwise norm convergence. When \mathcal{A} is singly generated and $\mathcal{B} = B(\ell^2)$, an affirmative answer was given in [4]; we extend this to the case when \mathcal{A} is separable. We also give an affirmative answer when \mathcal{B} is a von Neumann algebra and \mathcal{A} is AF or homogeneous; if \mathcal{B} is finite \mathcal{A} need only be ASH.

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