

SOME GENERAL QUADRATIC NUMERICAL RADIUS INEQUALITIES FOR THE OFF-DIAGONAL PARTS OF 2×2 BLOCK OPERATOR MATRICES

RONGFANG LI, DEYU WU* AND ALATANCANG CHEN

Abstract. In this paper, the elementary properties for the quadratic numerical radius are introduced. Furthermore, some general quadratic numerical radius inequalities for the off-diagonal parts of 2×2 block operator matrices are studied. These inequalities are based on the generalized mixed Schwarz inequality, Young inequality and Jensen inequality.

Mathematics subject classification (2020): 47A12.

Keywords and phrases: Bounded block operator matrix, quadratic numerical range, quadratic numerical radius, mixed Schwarz inequality, Young inequality, Jensen inequality.

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