

DAVIS-WIELANDT-BEREZIN RADIUS INEQUALITIES VIA DRAGOMIR INEQUALITIES

RAMIZ TAPDIGOGLU, MEHMET GÜRDAL*, NAJLA ALTWAIJRY AND NUR SARI

Abstract. We consider operator A on the reproducing Kernel Hilbert space $\mathcal{H} = \mathcal{H}(\Omega)$ over some set Ω with the reproducing kernel $\mathcal{K}_\lambda(z) = \mathcal{K}(z, \lambda)$ and define Davis-Wielandt-Berezin radius $\eta(A)$ by the formula

$$\eta(A) := \sup \left\{ \sqrt{|\tilde{A}(\lambda)|^2 + \|A\mathcal{K}_\lambda\|^4} : \lambda \in \Omega \right\},$$

where \tilde{A} is the Berezin symbol of A defined by $\tilde{A}(\lambda) := \langle A\widehat{\mathcal{K}}_\lambda, \widehat{\mathcal{K}}_\lambda \rangle$, $\lambda \in \Omega$, where $\widehat{\mathcal{K}}_\lambda = \frac{\mathcal{K}_\lambda}{\|\mathcal{K}_\lambda\|_{\mathcal{H}}}$ is the normalized reproducing kernel of \mathcal{H} . We prove several inequalities for this new quantity $\eta(A)$ involving known Dragomir inequalities. Some other Berezin number inequalities are also proved.

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