

MAXIMAL NUMERICAL RANGE AND QUADRATIC ELEMENTS IN A C^* -ALGEBRA

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Abstract. In this paper, we give a description of the maximal numerical range of a hyponormal element and a characterization of a normaloid element in a C^* -algebra. We also give an explicit formula for the maximal numerical range of a quadratic operator acting on a complex Hilbert space. As a consequence, we determine the maximal numerical range of a rank-one operator.

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