

## CANONICAL DECOMPOSITION FOR DISSIPATIVE LINEAR RELATIONS

JOSUÉ I. RIOS-CANGAS

*Abstract.* This work presents two ways of decomposing dissipative linear relations based on the fundamental decompositions for contractions by Sz. Nagy-Foiaş-Langer and von Neumann-Wold. The invariant subspaces for contractions are treated using a minor variation of the Cayley transform. The obtained decompositions allow one to separate the selfadjoint and completely nonselfadjoint parts of a dissipative relation.

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