

SURGERY OF FRAMES IN HILBERT SPACES

DONGWEI LI AND JING JIANG

Abstract. Frames which are tight or full spark might be considered optimally conditioned in applications. This leads to the question of perfect preconditioning of frames. In this paper, we consider the surgery of frames such that given frames can be manipulated to tight or full spark frames by removing and adding some elements. We give a necessary and sufficient condition such that a (r, k) -surgery on a frame results in a tight frame. We also provide a necessary and sufficient condition such that a $(1, k)$ -surgery on a tight frame resulting in a tight frame with same bound. Finally, we characterize that a (r, k) -surgery on a frame resulting in a full spark frame is possible. We obtain a necessary and sufficient condition such that a (r, k) -surgery on a given frame results in a full spark frame.

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