

ON THE SPECTRUM OF COMPLEX SYMMETRIC TOEPLITZ OPERATORS

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Abstract. In this paper we focus on the spectrum of Toeplitz operators T_ϕ on the Hardy space H^2 of the unit disk \mathbb{D} when ϕ is not continuous and especially when T_ϕ is a complex symmetric operator. Moreover, we show a necessary condition for T_ϕ to be complex symmetric, for ϕ belongs to subalgebra $H^\infty + \mathcal{C}(\mathbb{T})$ of L^∞ and we present a case of the continuity of the spectral function restricted to the space of Toeplitz operators.

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