ON THE SIMILARITY OF COMPLEX SYMMETRIC OPERATORS TO PERTURBATIONS OF RESTRICTIONS OF NORMAL OPERATORS

SERGEY M. ZAGORODNYUK

Abstract. In this paper we consider a problem of the similarity of complex symmetric operators to perturbations of restrictions of normal operators. For a subclass of cyclic complex symmetric operators in a finite-dimensional Hilbert space we prove the similarity to rank-one perturbations of restrictions of normal operators. The main tools are a truncated moment problem in $\mathbb{C}$, and some objects similar to objects from the theory of spectral problems for Jacobi matrices.

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