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EXPANSIVE OPERATORS WHICH ARE POWER BOUNDED OR ALGEBRAIC

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Abstract. Given Hilbert space operators $P,T \in B(\mathscr{H}), P \geqslant 0$ invertible, T is (m,P)-expansive (resp., (m,P)-isometric) for some positive integer m if $\triangle^m_{T^*,T}(P) = \sum_{j=0}^m (-1)^j \binom{m}{j} T^{*j}PT^j \leqslant 0$ (resp., $\triangle^m_{T^*,T}(P) = 0$). Power bounded (m,P)-expansive operators, and algebraic (m,I)-expansive operators have a simple structure. A power bounded operator T is an (m,P)-expansive operator if and only if it is a $C_{1\cdot}$-operator such that $\|QTx\| = \|Qx\|$ (i.e., T is Q-isometric) for some invertible positive operator Q. If, instead, T is an algebraic (m,I)-expansive operator, then either the spectral radius $r(T)$ of T is greater than one or T is the perturbation of a unitary by a nilpotent such that T is $(2n-1,I)$-isometric for some positive integers $m_0 \leqslant m$, m_0 odd, and $n \geqslant \frac{m_0+1}{2}$.

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REFERENCES

[1] J. AGLER, *Hypercontractions and subnormality*, J. Operator Theory **13** (1985), 203–217.
[2] J. AGLER AND M. STANKUS, *m-Isometric transformations of Hilbert space I*, Integr. Equat. Oper. Theory **21** (1995), 383–420.
[3] T. ANDO, *Topics on Operator Inequalities*, Division of Applied Mathematics, Research Institute of Applied Electricity, Hokkaido University, Sapporo Japan (1978).
[4] A. ATHAVALE, *On completely hyperexpansive operators*, Proc. Amer. Math. Soc. **124** (1996), 3745–3752.
[5] F. BAYART, *m-isometries on Banach Spaces*, Math. Nachr. **284** (2011), 2141–2147.
[6] F. BOTELHO AND J. JAMISON, *Isometric properties of elementary operators*, Linear Alg. Appl. **432** (2010), 357–365.
[7] T. BERMÚDEZ, A. MARTINÓN AND J. N. NODA, *Products of m-isometries*, Linear Alg. Appl. **408** (2013) 80–86.
[8] B. P. DUGGAL, *Tensor product of n-isometries*, Linear Alg. Appl. **437** (2012), 307–318.
[9] B. P. DUGGAL AND C. S. KUBRUSLY, *Power bounded m-left invertible operators*, Linear and Multilinear Algebra, **69** (3) (2021), 515–525, doi:10.1080-03081087.2019.16044623.
[10] R. G. DOUGLAS, *On majorization, factorization and range inclusion of operators on Hilbert space*, Proc. Amer. Math. Soc. **17** (1966), 413–415.
[11] G. EXNER, I. B. JUNG AND C. LI, *On k-hyperexpansive operators*, J. Math. Anal. Appl. **323** (2006), 569–582.
[12] C. GU, *Structure of left n-invertible operators and their applications*, Studia Math. **226** (2015), 189–211.
[13] C. GU, *On (m,P)-expansive and (m,P)-contractive operators on Hilbert and Banach spaces*, J. Math. Anal. Appl. **426** (2015), 893–916.
[14] S. JUNG, Y. KIM, E. KO AND J. E. LEE, *On (A,m)-expansive operators*, Studia Math. **213** (2012), 3–23.

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[15] L. KÉRCHY, *Isometric asymptotes of power bounded operators*, Indiana Univ. Math. J. **38** (1989), 173–188.
[16] D. KOEHLER AND P. ROSENTHAL, *On isometries of normed linear spaces*, Studia Math. bf 36 (1970), 213–216.
[17] C. S. KUBRUSLY, *An Introduction to Models and Decompositons in Operator Theory*, Springer Science and Business Media, LLC(1997).
[18] C. S. KUBRUSLY AND B. P. DUGGAL, *Asymptotic limits, Banach limits and Cesaro means*, Advances in Mathematical Sciences and Applications **29** (1) (2020), 145–170.
[19] TRIEU LE, *Algebraic properties of operator roots of polynomials*, J. Math. Anal. Appl. **421** (2015), 1238–1246.