

## DISTINGUISHED SUBSPACES OF TOPELITZ OPERATORS ON $N_\varphi$ -TYPE QUOTIENT MODULES

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*Abstract.* In this paper, we show that there always exists reducing subspace  $M$  for  $S_{\psi(z)}$  such that the restriction of  $S_{\psi(z)}$  on  $M$  is unitarily equivalent to the Bergman shift when  $\psi(z)$  is a finite Blaschke product. Moreover, we will show that only if  $\psi(z)$  is a finite Blaschke product can  $S_{\psi(z)}$  has distinguished reducing subspaces. We also give the form of these distinguished reducing subspaces when  $\psi(z)$  is a finite Blaschke product. Finally, we show that every non-trivial minimal reducing subspace  $S$  of  $S_{\psi(z)}$  is orthogonal to the direct sum of all distinguished subspaces when  $S$  is not a distinguished subspace of  $S_{\psi(z)}$ .

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