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UNIT VECTORS IN FULL HILBERT $C(Z)$–MODULES

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Abstract. In this paper, we show that full Hilbert $C(Z)$-modules, where Z is a compact Hausdorff space may fail to have unit vectors. We also show that while real Hilbert $C_{\mathbb{R}}(Z)$-modules may not have unit vectors, their complexifications as (complex) Hilbert $C(Z)$-modules may have unit vectors. In particular, we prove that: (i) the unit vectors in full Hilbert $C(Z)$-modules are precisely the extreme points of their unit balls; (ii) the extreme and the exposed points of the unit ball of full Hilbert $C(Z)$-modules with unit vectors coincide as Z has a diffuse measure; otherwise, their unit balls have no exposed points.

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