

THE FOURIER TRANSFORM OF ANISOTROPIC HARDY SPACES WITH VARIABLE EXPONENTS AND THEIR APPLICATIONS

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Abstract. Let A be an expansive dilation on \mathbb{R}^n , and $p(\cdot) : \mathbb{R}^n \rightarrow (0, \infty)$ be a variable exponent function satisfying the globally log-Hölder continuous condition. Let $\mathcal{H}_A^{p(\cdot)}(\mathbb{R}^n)$ be the variable anisotropic Hardy space introduced by Liu [15]. In this paper, the authors obtain that the Fourier transform of $f \in \mathcal{H}_A^{p(\cdot)}(\mathbb{R}^n)$ coincides with a continuous function F on \mathbb{R}^n in the sense of tempered distributions. As applications, the authors further conclude a higher order convergence of the continuous function F at the origin and then give a variant of the Hardy-Littlewood inequality in the setting of anisotropic Hardy spaces with variable exponents.

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