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ON OPERATORS SATISFYING $T^*(T^{*2}T^2)^pT \geqslant T^*(T^2T^{*2})^pT$

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Abstract. An operator $T \in B(H)$ is called square- p -quasihyponormal if

$$T^*(T^{*2}T^2)^pT \geqslant T^*(T^2T^{*2})^pT \quad \text{for} \quad p \in (0,1],$$

which is a further generalization of normal operator. In this paper, we give a sufficient condition for an injective square- p -quasihyponormal operator to be self-adjoint, and we obtain that every square- p -quasihyponormal operator has a scalar extension. As a consequence, we prove that if T is a quasiaffine transform of square- p -quasihyponormal, then T satisfies Weyl's theorem. Finally some examples are presented.

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