

## ON MULTIPLICITIES OF EIGENVALUES OF A SPECTRAL PROBLEM ON A PROLATE TREE

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*Abstract.* It is known that there exists an ordered vector majorizing any ordered vector of possible multiplicities of eigenvalues of the star-patterned matrix. The same situation occurs in the problem of small transverse vibrations of a star graph of Stieltjes strings. We show that for a certain class of prolate trees of Stieltjes strings there exists an ordered vector majorizing any ordered vectors of possible eigenvalue multiplicities of spectral problems on such trees.

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