

KOTANI THEORY FOR ERGODIC BLOCK JACOBI OPERATORS

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Abstract. We extend the so-called Kotani Theory for a particular class of ergodic block Jacobi operators defined in $l^2(\mathbb{Z}; \mathbb{C}^l)$ by the law $[H_\omega \mathbf{u}]_n := D^*(T^{n-1} \omega) \mathbf{u}_{n-1} + D(T^n \omega) \mathbf{u}_{n+1} + V(T^n \omega) \mathbf{u}_n$, where $T : \Omega \rightarrow \Omega$ is an ergodic automorphism in the measure space (Ω, ν) , the map $D : \Omega \rightarrow GL(l, \mathbb{R})$ is bounded, and for each $\omega \in \Omega$, $D(\omega)$ is symmetric and $D^{-1}(\omega)$ is bounded. Namely, it is shown that for each $r \in \{1, \dots, l\}$, the essential closure of $\mathcal{L}_r := \{x \in \mathbb{R} \mid \text{exactly } 2r \text{ Lyapunov exponents of } A_x \text{ are zero}\}$ coincides with $\sigma_{ac, 2r}(H_\omega)$, the absolutely continuous spectrum of multiplicity $2r$, where A_x is a Schrödinger-like cocycle induced by H_ω . Moreover, if $k \in \{1, \dots, 2l\}$ is odd, then $\sigma_{ac, k}(H_\omega) = \emptyset$ for ν -a.e. $\omega \in \Omega$. We also provide a Thouless formula for such class of operators.

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