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ON SOME FUGLEDE–KADISON DETERMINANT INEQUALITIES OF OPERATOR MEANS

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Abstract. Let $\mathscr{M}$ be a finite von Neumann algebra with finite trace τ. We extend some important matrix determinant inequalities, studied by Lin, Ghabries, Abbas, Mourad and Assi, to the Fuglede-Kadison determinant of τ-measurable operators in the noncommutative algebra $L_{\log_+}(\mathscr{M})$. Some Fuglede-Kadison determinant inequalities are established in $L_{\log_+}(\mathscr{M})$ with different forms to the matrix case.

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