

UNIVERSALLY SYMMETRIC NORMING OPERATORS ARE COMPACT

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Abstract. We study a specific family of symmetric norms on the algebra $\mathcal{B}(\mathcal{H})$ of operators on a separable infinite-dimensional Hilbert space. With respect to each symmetric norm in this family the identity operator fails to attain its norm. Using this, we generalize one of the main results from [8]; the hypothesis is relaxed, and consequently, the family of symmetric norms for which the result holds is extended.

We introduce and study the concepts of “universally symmetric norming operators” and “universally absolutely symmetric norming operators” on a separable Hilbert space. These refer to the operators that are, respectively, norming and absolutely norming, with respect to every symmetric norm on $\mathcal{B}(\mathcal{H})$. We establish a characterization theorem for such operators and prove that these classes are identical, and that they coincide with the class of compact operators. In particular, we provide an alternative characterization of compact operators on a separable infinite-dimensional Hilbert space.

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