

IMPROVED RATE OF APPROXIMATION BY MODIFICATION OF BASKAKOV OPERATOR

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Abstract. The optimal order of approximation, $|L_n f(x) - f(x)|$ of a linear positive operator $L_n f(x)$ is $1/n$ and can not be improved however smooth the function may be. We remove the positivity of the Baskakov operator $V_n(f;x)$ and introduce its three variants $V_n^{M,i}(f;x)$, $i = 1, 2, 3$. We prove that the rates of approximation by these operators are improved from the linear order $1/n$ to quadratic order $1/n^2$ and then to cubic order $1/n^3$ for sufficiently smooth functions.

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