

## UNITARY, SELF-ADJOINT AND $\mathcal{J}$ -SYMMETRIC WEIGHTED COMPOSITION OPERATORS ON FOCK-SOBOLEV SPACES

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**Abstract.** In this paper, we characterize the boundedness and compactness for weighted composition operators on the Fock-Sobolev space  $\mathcal{F}^{p,m}(\mathbb{C}^n)$ ,  $0 < p < \infty$ . We prove that no nontrivial unitary or self-adjoint weighted composition operators exist on  $\mathcal{F}^{2,m}(\mathbb{C}^n)$  when  $m \geq 1$ . As an application, we also prove that there exist only trivial  $\mathcal{J}$ -symmetric weighted composition operators on  $\mathcal{F}^{2,m}(\mathbb{C}^n)$  when  $m \geq 1$ .

**Mathematics subject classification (2020):** 30H20, 47B15, 47B33.

**Keywords and phrases:** Fock-Sobolev space, weighted composition operator, unitary operator, self-adjoint, complex symmetry.

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