Operators and Matrices
Volume 17, Number 1 (2023), 1–23
doi:10.7153/oam-2023-17-01

APPROXIMATE EQUIVALENCE IN VON NEUMANN ALGEBRAS

QIHUI LI, DON HADWIN AND WENJING LIU

Abstract. Suppose $\mathcal{A}$ is a separable unital ASH C*-algebra, $\mathcal{M}$ is a sigma-finite II_∞ factor von Neumann algebra, and $\pi, \rho : \mathcal{A} \to \mathcal{M}$ are unital $*$-homomorphisms such that, for every $a \in \mathcal{A}$, the range projections of $\pi(a)$ and $\rho(a)$ are Murray von Neuman equivalent in $\mathcal{M}$. We prove that π and ρ are approximately unitarily equivalent modulo $\mathcal{K}_\mathcal{M}$, where $\mathcal{K}_\mathcal{M}$ is the norm closed ideal generated by the finite projections in $\mathcal{M}$. We also prove a very general result concerning approximate equivalence in arbitrary finite von Neumann algebras.

Mathematics subject classification (2020): 47C15 (46L10).

Keywords and phrases: Approximate equivalence, semifinite von Neumann algebra, ASH C*-algebra, center-valued trace, $\mathcal{M}$-rank.

REFERENCES

[1] W. ARVESON, *An invitation to C*-algebras*, Graduate Texts in Mathematics, No. 39, Springer-Verlag, New York-Heidelberg, 1976.
[2] A. CIUPERCA, T. GIORDANO, P. W. NG, AND Z. NIU, *Amenability and uniqueness*, Adv. Math. 240 (2013) 325–345.
[3] K. R. DAVIDSON, *C*-algebras by example*, Fields Institute Monographs, 6, Amer. Math. Soc., Providence, RI, 1996.
[4] H. DING AND D. HADWIN, *Approximate equivalence in von Neumann algebras*, Sci. China Ser. A 48 (2005), no. 2, 239–247.
[5] S. WEN, J. FANG AND R. SHI, *Approximate equivalence of representations of AF algebras into semifinite von Neumann algebras*, Oper. Matrices 13 (2019), no. 3, 777–795.
[6] J. GLIMM, *Type I C*-algebras*, Ann. Math. 73 (1961) 572–612.
[7] D. HADWIN, *Nonseparable approximate equivalence*, Trans. Amer. Math. Soc. 266 (1981), no. 1, 203–231.
[8] D. HADWIN, W. LI, W. LIU, AND J. SHEN, *A characterisation of tracially nuclear C*-algebras*, Bull. Aust. Math. Soc. 100 (2019), no. 1, 119–128.
[9] D. HADWIN AND RUI SHI, *A note on representations of commutative C*-algebras in semifinite von Neumann algebras*, Oper. Matrices 12 (2018), no. 4, 1129–1144.
[10] P. R. HALMOS, *Ten problems in Hilbert space*, Bull. Amer. Math. Soc. 76 (1970) 887–933.
[11] R. V. KADISON AND J. RINGROSE, *Fundamentals of the Theory of Operator Algebras*, Vol. 2: Advanced Theory (Graduate Studies in Mathematics, Vol. 16), Academic Press, 1983.
[12] Q. LI, J. SHEN, R. SHI, *A generalization of Voiculescu's theorem for normal operators to semifinite von Neumann algebras*, Adv. Math. 375 (2020) 107347.
[13] D. SHERMAN, *Unitary orbits of normal operators in von Neumann algebras*, J. Reine Angew. Math. 605 (2007), 95–132.
[14] R. SHI AND JUNHAO SHEN, *Approximate equivalence of representations of AH algebras into semifinite von Neumann algebras*, arXiv:1805.07236, 2018.
[15] D. V. VOICULESCU, *A non-commutative Weyl-von Neumann theorem*, Rev. Roumaine Math. Pures Appl. 21 (1976), no. 1, 97–113.
[16] N. E. WEGGE-OLSEN, *K-theory and C*-algebras, A friendly approach*, Oxford Science Publications. The Clarendon Press, Oxford University Press, New York, 1993.

Paper OaM-17-01