

ON ITERATED DISCRETE HARDY TYPE OPERATORS

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Abstract. The paper discusses a new iterated discrete inequality of Hardy type involving an operator with some matrix kernel. Under certain conditions on this matrix kernel, the given inequality is characterized.

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