

## DUALITY OF GENERALIZED HARDY AND BMO SPACES ASSOCIATED WITH SINGULAR PARTIAL DIFFERENTIAL OPERATOR

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*Abstract.* First we define and study the generalized bounded mean oscillation space  $\mathfrak{B}mo_\alpha$  associated with the Riemann-Liouville operator  $\mathcal{R}_\alpha$ . Next we prove the duality between  $\mathfrak{B}mo_\alpha$  and the generalized Hardy space  $\mathcal{H}_\alpha^1$  associated with  $\mathcal{R}_\alpha$ .

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