

A CONSTRUCTIVE PROOF OF A NONCOMMUTATIVE FEJÉR–RIESZ THEOREM

PALAK ARORA

Abstract. In this paper, we present a constructive proof of Popescu’s Fejér-Riesz theorem for noncommuting polynomials representing nonnegative “multi-Toeplitz” operators.

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