

ON SOLVABILITY OF VOLTERRA–HAMMERSTEIN INTEGRAL EQUATIONS IN TWO VARIABLES COORDINATEWISE CONVERGING AT INFINITY

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Abstract. We will investigate the existence of solutions to an infinite system of nonlinear integral equations in two variables of the Volterra–Hammerstein type. The approach we take in our research relates to the construction of an appropriate measure of noncompactness in the space of functions defined, continuous, and bounded on $\mathbb{R}_+ \times \mathbb{R}_+$ with values in the space ℓ_∞ endowed with the standard supremum norm and created by function sequences that are coordinatewise converging to proper limits at infinity. Our research is illustrated with an example.

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