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THE STABILITY OF PROPERTY (gt) UNDER PERTURBATION AND TENSOR PRODUCT

MOHAMMAD H. M. RASHID AND MUNEO CHŌ

Abstract. An operator T acting on a Banach space $\mathscr{X}$ obeys property (gt) if the isolated points of the spectrum $\sigma(T)$ of T which are eigenvalues are exactly those points λ of the spectrum for which $T-\lambda$ is an upper semi-B-Fredholm with index less than or equal to 0. In this paper we study the stability of property (gt) under perturbations by finite rank operators, by nilpotent operators and, more generally, by algebraic operators commuting with T. Moreover, we study the transfer of property (gt) from a bounded linear operator T acting on a Banach space $\mathscr{X}$ and a bounded linear operator S acting on a Banach space $\mathscr{Y}$ to their tensor product $T\otimes S$.

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