

## UNCERTAINTY INEQUALITIES FOR WEIGHTED SPACES OF ANALYTIC FUNCTIONS ON THE UNIT DISK

FETHI SOLTANI

*Abstract.* In this paper we establish uncertainty inequality of Heisenberg type for Hardy space  $\mathcal{H}$ , Dirichlet space  $\mathcal{D}$  and Bergman space  $\mathcal{B}$ , respectively. Next, we introduce a weighted Hardy space  $\mathcal{H}_\beta$ . This space which gives a generalization of some Hilbert spaces of analytic functions on the unit disk like, the Hardy space  $\mathcal{H}$ , the Dirichlet space  $\mathcal{D}$  and the Bergman space  $\mathcal{B}$ , it plays a background to our contribution. Especially, we study the derivative operator  $D$  and its adjoint operator  $L_\beta$  on  $\mathcal{H}_\beta$ , and we deduce a general uncertainty inequality of Heisenberg type for this space.

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